Multiphysics modeling of warm-air drying of potatoes slices

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Abstract: Potato is a hygroscopic porous material and drying is a processing operation applied to this product. Modeling drying is a complex task since interaction between phases and variations in thermal properties changes during time. In this work we solve a model to simulate the drying of potatoes slices. The model considers both the transport of free and vapor water by applying a mechanistic approach. The critical moisture point (CMP) was considered, since it is a transition zone and it represents the point where water saturation is near from cero and hygroscopic domain begins. The CMP divides the hygroscopic and non-hygroscopic domain of the material. At values above CMP free water is removed, and bellow CMP bound water and water vapor are removed. On one hand, to simulate free water transport we take into account the capillary diffusivity term, and by other hand, to simulate water vapor evacuation we consider the desorption isotherm of potatoes. The sorption isotherm depends on local moisture content and temperature. The model solves the primary unknown's moisture content, temperature and dry-air density.

Keywords: Potato, drying, capillary diffusivity

1. Introduction

Drying is the most frequent operation for food preservation. However, drying foods follow important alterations as shrinkage, degradation of nutrients, browning, etc. Mathematical models are important to get a deeper understanding about drying. Models allow to optimize the energy consumption, the quality of product and to reduce the operation time. A mathematical model is a mathematical analog of the physical reality, describing the properties and features of a real system in terms of mathematical variables and operations. Mathematical models can be classified into physics-based and observation-based models [1]. Drying is a multiphase phenomena, interesting for a wide kind of materials [2] [3] [4] [5] [6]. In this work we are interested about a physic-based model since it is based on the universal physical laws that describe the liquid and vapor transport phenomena. Physics-based models are also validated against experimental data, but in physics-based models the experimental data do not have to exist before the model. [7] explains more about these kind models.

The mathematical models can predict the moisture evolution and moisture patterns in materials during drying. In this way, models facilitate the understanding of moisture mobility. With theoretical results we can improve practical problems.

The potato is a homogeneous material that has been widely studied due to its low cost and its availability. There are many models for drying; however, most are based on diffusion mechanism, which limits to much the understanding about moisture mobility in the capillary region [2] [8] [9] [10]. Other works has applied models to simulate drying of heterogeneous and complex materials [11]. Physics-based models need thermo-physical properties which are a function of temperature and moisture content, then a difficulty is to obtain those data, nevertheless lab facilities and modern modeling technics can offer innovative solutions to drying.

In recent years, innovative technology and software has been an effective tool to drying. and even more important, the use of computers and software has change the way we think, in fact computer objects has become more explicity designed to have cognitive effects [12].

In this work, experimental data and theoretical model are compared to explain simultaneous transport mechanims occurring in drying. A physics-based model was solved by coupling equations in Comsol Multiphysics EDP mode.

2. Methods

2.1 Material:

Potato (variety Alpha) was obtained from a local supermarket and stored at 3–4°C. Slices from fresh potatoes (50 mm diameter, 20mm height) were prepared for drying. A blanching treatment in hot water at 70ºC during 1 minute was applied in order to inhibit enzymatic reactions. The initial moisture content was determined by using the standard Oven Method [13].

2.2 Equipment:

The trials were performed in a dryer tunnel; the dryer was equipped with a data acquisition system. Data was logged every 2 minutes. Two temperatures were measured with J-type thermocouples (Figure 1), and the surface temperature was measured with an infrared thermometer (EXTECH model 42560) located on the top of the tunnel.

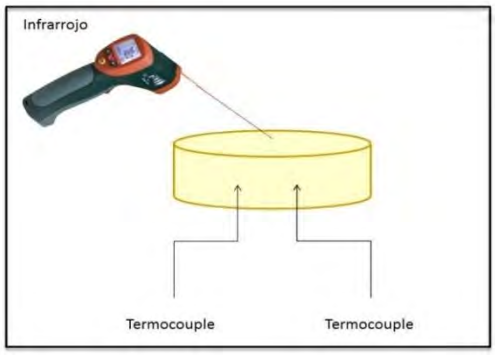

Figure 1. Measurement of internal and surface temperatura of the material.

2.3 Drying process:

Unifactorial design was used with variable velocity air with 2 levels (1.0 and 2.2 m/s). All experiments were replicate 4 times. The temperature and humidity inside the dryer were constant. Drying was controlled at 60°C and 30% RH. Data were standardized by computing CH/CH_{max} and $(dW/dt)/(dW/dt)_{max}$.

3. Governing Ecuations

[14] proposed thermodynamic fundamentals to model drying. In this work the Whitaker's approach was used. A representative volume element represents three phases whose coexist and are at its thermodynamic equilibrium. A mathematical model was adapted in COMSOL multiphysics by adopting the experimental configuration. Partial differential equations were written for moisture content, temperature, and dry-air conservation. The model captures both the transport of liquid water in the capillary domain, and the transport of water vapor and air. The model is based on the following assumptions:

- Gravitational effects are negligible
- Temperature and pressure in the dryer are homogeneous
- The gas phase is water vapor, which behaves as an ideal gas.
- Thermodynamic equilibrium, so average temperatures for each phase are the same, and partial vapor pressure is at equilibrium.
- The material is homogeneous and non-deformable, so that shrinkage is negligible
- The heat and mass transfer takes place only by the upper and lower faces, and disregards the transfer on the small walls (Figure 2).

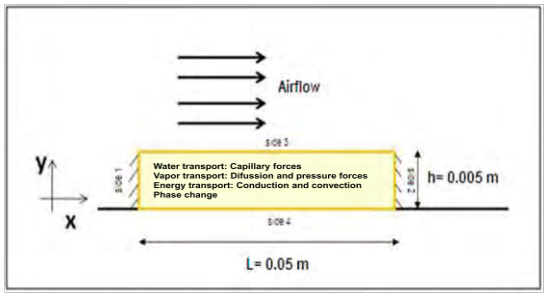

Figure 2. Physical Model for potato

4. Theory

Continuity equations for each phase by considering the change of phase is:

Liquid Water: $$\frac{\partial \rho_l}{\partial t} + \nabla \cdot \left(\rho_l^l \overrightarrow{V_l}\right) = -I \quad (1)$$

Vapor Water: $$\frac{\partial \rho_v}{\partial t} + \nabla \cdot \left(\rho_v^g \overrightarrow{V_v}\right) = I \quad (2)$$

Then, the moisture transport equation is the time-dependent equation:

$$\frac{\partial W}{\partial t} + \nabla \cdot \left\{\frac{1}{\rho_s}\left(\rho_l \overrightarrow{V_l} + \rho_v^g \overrightarrow{V_v}\right)\right\} = 0 \quad (3)$$

where W is the moisture content (kg water/kg dry mass); ρ_s is solid density (kg/m^3); $\rho_l\vec{V_l}$ represents the liquid water flux and $\rho_v^g\vec{V_v}$ the vapor water flux.

The energy general balance takes into account the mass flux for each phase. This term is affected by the heat capacity to represent the energy transported for each phase, the energy equation is:

$$\rho Cp\frac{\partial T}{\partial t} + [(\rho_l\vec{V_l}Cp_l + \rho_a^g\vec{V_a}Cp_a + \rho_v^g\vec{V_v}Cp_v)]\nabla T - \nabla\cdot(\lambda\cdot\nabla T) = 0 \quad (4)$$

where Cp is heat capacity (J/kg K) for liquid phase (l), vapor (v) and air (a) respectively; T is temperature (K) and λ is thermal conductivity tensor (W/m^2 K) (see the Table 1). Equations (3) and (4) are a non-lineal partial differential equations system, because of material parameters depends on local moisture content and temperature.

In the first minutes of drying, liquid water flow is carried out by capillary forces. Free water evacuation is written by using the concept of *Capillary Diffusivity*, which involves all phenomena that taking place within the capillarity domain [4].

$$\rho_l\vec{V_l} = -D_c\cdot[\nabla W] \quad (5)$$

This expression has been used for a few authors [3] [4] [5] [15]; the capillary diffusivity coefficient is function of moisture content:

$$D_c = 1X10^{-8}\exp(-2.8 + 2.0\,W) \quad (6)$$

For the mobility of water vapor, a pressure gradient is considered; the expression translates the vapor transport as mass vapor evacuated by both a pressure and concentration gradient:

$$\rho_v^g\overrightarrow{V_v} = \rho_v^g\frac{k.k_{rg}}{g}\cdot\nabla P_g^g - \rho_g^g D_{eff}\cdot\nabla C_v^g \quad (7)$$

where k is the intrinsic permeability (m^2); k_{rg} the gas relative permeability (m^2); μ_g the gas viscosity (Pa s) and D_{eff} is the effective diffusion coefficient (m^2/s). The term C_v^g is the mass concentration of water vapor in the gaseous phase. An equivalent expression is used for the dry-air.

ρ_v^g was computed from the ideal gas law, $PV=nRT$, where P_v^g is vapor pressure in the gaseous phase, which is function of both the water activity a_w, and saturation vapor pressure P_v^{sat}:

$$P_v^g = a_w.P_v^{sat} \quad (8)$$

Water activity is a thermodynamic concept, defined by [16] as the ratio of the fugacity, f, of a substance and its fugacity, f_o, at a given temperature. Water activity is a very useful factor in assessing mobility controlled processes in foods because it controls the water content in the various food components, and the water content (because of plasticizing action) has a dominating effect on mobility of hydrophilic food components [17]. In COMSOL we have used the tool "insert table" to interpolate the isotherm, and then the equilibrium moisture content is governed by this desorption isotherm. At values above the free water is removed by capillary flow; in the hygroscopic domain bound water is removed by diffusion [18]. In drying we are often concerned not only with local concentration gradients but also with local activity gradients since they control the fluxes. Furthermore activities determine the local water concentrations which in turn determine local mobility, local rates of reaction, and local transition. Porosity (Φ) is the ratio between the volume of voids and total volume. Table 1 shows the equations used for these parameters.

Tabla 1. Definitions of parameters

$a_w = exp(-exp(1.7566-9.577x10^{-3}T_\infty)W^{0.9764})$
$P_v^{sat} = exp\left[20.90006 - \frac{5204.9}{T}\right]$
$S_w = \frac{\rho_s(W-W_{crit})}{\phi.\rho_l}$ cuando $W > W_{crit}$
$Deff = \left[1.0418x10^{-5}\exp\left(\frac{-25.77}{8.314x10^{-3}\,T}\right)\right]$
$\lambda = 0.276 + 0.293\log(W)$

(a)= [19] (b)= [9] (c)= [10]

The initial conditions establish that at t=0, the initial moisture content and temperature are those measured in the laboratory. Dirichlet conditions were applied to simulate the heat transfer:

$$Ts = \boldsymbol{n}\cdot(\nabla T) \quad (9)$$

Where *Ts* incorporates the values of temperature on the material surface. Equation (9) represents the imposed temperature at the surface. A Neumann boundary condition was imposed for mass transfer that includes dry-air and water vapor density:

Mass: $-k_m(\rho_{v\infty} - \rho_v^g) = -\boldsymbol{n} \cdot \left(-D\nabla C_v^g\right)$

Air: $k_a(\rho_{a\infty} - \rho_a^g) = -\boldsymbol{n} \cdot \left(-D\nabla C_a^g\right)$

Where $\rho_{v\infty}$ and $\rho_{v\infty}$ are the dry-air and water vapor density within drying medium respectively.

5. Numerical solution

The mathematical model was solved in COMSOL Multiphysics 3.4©. The UMFPACK solver was used since it solves a linear system Ax = b or xA = b with a sparse square (says n x n) real or complex matrix. A LU factorization of the matrix is firstly computed. A 2D configuration was adopted with a 2nd-order Lagrange elements. A rectangular geometry was created which represents the material slice (5 cm diameter and 0.5 cm height). The simulation domain consisted of 296 triangular elements. The time step used was 100 seconds for a simulation time of 30 000 seconds [0:100:30000].

The assigned values for the relative and absolute tolerance are 0.1 and 0.01 respectively. The material parameters and properties were incorporated and the EDP model was adapted by the general formulation in COMSOL. The moisture content and dry-air density were the primary unknowns. Heat transport equation was solved by using the coefficients formulation.

6. Experimental Results and Discussion

Temperature and moisture content evolution are displayed in Figure 3 and 4. The capillary domain governed the system to 0.385 kg water/kg dry mass, where moisture internal gradients were observed and beging hygroscopic domain.

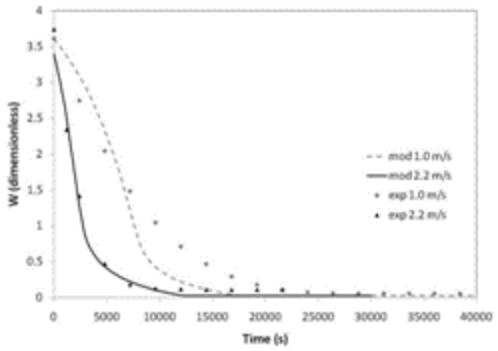

Figure 3. Experimental and simulated drying kinetics to 1.0 and 2.2 m/s.

It is interesting to see the temperature evolution at different air-velocity (Figure 4). In fact at 1 m/s the evaporation is slower than at 2 m/s. At 1 m/s we identify several periods, between them the constant drying period. Nevertheless at 2 m/s the material heating evolution show a typical behavior.

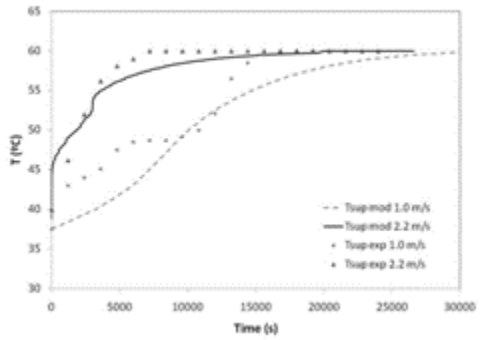

Figure 4. Temperature evolution at 1.0 y 2.2 m/s

In Figure 5, we show the moisture distribution at high and low air-velocity after two hours of drying.

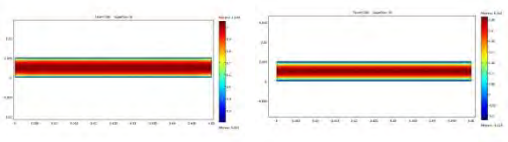

Figure 5. Moisture distribution after 2 hours of drying.

The deviation between the experimental and simulated curves was attributed to thermo-physical properties, since potatoes are biological materials and its properties changes significantly.

Error was computed by a minimization of quadratic error. The computed error for each air-flow condition was 0.29 and 0.12 (kg water/kg dry mass), for 1.0 and 2.2 m/s respectively.

7. Conclusions

We have solved a model to study drying of potatoes slices. Theoretical drying kinetics and computed temperature evolution give some confidence of the proposed solution. Deviations are attributed to thermo-physical variation of the material. The moisture content evolution is well simulated, and temperature show two behaviors depending on air-flow velocity.

8. References

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